

Schwinger-Dyson Study for Walking/Conformal Dynamics with IR Cutoffs

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Abstract

Motivated by recent progress on many flavor QCD on a lattice, we investigate conformal/walking dynamics by using Schwinger-Dyson (SD) equation within an improved ladder approximation for two-loop running coupling. By numerically solving the SD equation, we obtain a pole mass m_p , pion decay constant f_π , and investigate the chiral symmetry breaking and mass anomalous dimension γ_m in the presence of IR cutoffs Λ_{IR} . We find that the chiral symmetry breaking is suppressed if IR cutoff Λ_{IR} becomes larger than the critical value near the dynamical mass ($\Lambda_{\text{IR}} \simeq m_D$). In the conformal phase the γ_m is strongly suppressed by IR cutoffs for $\Lambda_{\text{IR}} \simeq m_p$. We, then, obtain finite size hyperscaling (FSS) relation by adapting a linearized approximation for the SD equation, and examine the γ_m . The results offer valuable insight and suggestion for analyses in lattice gauge theories.

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I. INTRODUCTION

When a fermion multiplicity (flavor) exceeds a critical value $N_f = N_f^*$ in non-Abelian gauge theories, a quantum phase transition from a chiral broken phase to the conformal window is anticipated to take place. In the conformal window, strong coupling gauge theories are asymptotically-free in ultra violet (UV) region, while dominated by a non-trivial infrared fixed point (IRFP) in infrared (IR) region. The IRFP-conformal dynamics can be probed via a response to a small fermion mass (m_0) perturbation, which gives rise to a bound state with a mass gap (M_H) satisfying the conformal hyperscaling relation with a mass anomalous dimension γ_m . In the chiral broken phase near to the conformal window $N_f \lesssim N_f^*$, it is advocated that the system shows an approximate conformality (walking) with a “would-be” γ_m , which is of great interest in physics beyond the Standard Model [1]. In recent lattice studies, the precise determination of γ_m in many flavor QCD is reported, while there exist tensions among the values [2]. The discrepancy presumably originates to scale violations by UV/IR cutoffs (lattice spacing and finite box size) as well as a probe fermion mass. In this proceedings, we focus on the IR cutoff effects by using Schwinger-Dyson (SD) equation.

II. SCHWINGER-DYSON EQUATION ANALYSIS

We consider the SD equation for the fermion propagator $iS_F(q) \equiv [\not{p} - \Sigma(p^2)]^{-1}$ within improved ladder approximation in the Landau gauge. After integrating over angular degrees of freedom in a momentum space, the SD equation is expressed as

$$\Sigma(p^2) = m_0 + \int_{\Lambda_{\text{IR}}}^{\Lambda_{\text{UV}}} dq^2 \frac{3C_2[F]\alpha(p^2 + q^2)}{4\pi} \left[\frac{q^2}{p^2} \theta_{p^2 - q^2} + \theta_{q^2 - p^2} \right] \frac{\Sigma(q^2)}{q^2 + \Sigma^2(q^2)}, \quad (1)$$

where m_0 , θ_{p^2} , $\alpha(\mu) = g^2(\mu)/(4\pi^2)$, and $C_2[F]$ represent a bare fermion mass, step function, running coupling constant in the two-loop perturbation theory, and quadratic Casimir operator, respectively. The UV/IR cutoffs $\Lambda_{\text{UV/IR}}$ are introduced in the momentum space integral. For a given fermion mass m_0 and in the presence of $\Lambda_{\text{UV/IR}}$, we numerically solve the SD equation (1) and evaluate the physical pole mass m_p and pion decay constant f_π as

$$m_p \equiv \Sigma(p^2 = m_p^2), \quad f_\pi^2 = \frac{N_c}{4\pi^2} \int_0^{\Lambda_{\text{UV}}^2} dz \, z \frac{(1 - \frac{1}{4}z \frac{d}{dz})\Sigma^2(z)}{(z + \Sigma^2(z))^2}. \quad (2)$$

We take account of full momentum dependences of the two-loop running coupling without recourse to the usual step-function type approximation for the coupling. We then perform the hyperscaling fit analyses for the obtained numerical data similarly to analyses in lattice gauge theories. Moreover,

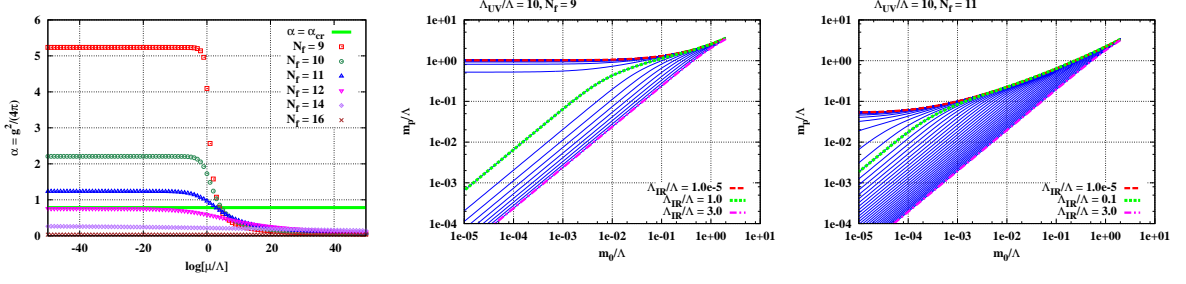


FIG. 1: Left: The running coupling for the various fermion multiplicity $9 \leq N_f \leq 16$ in the color $SU(N_c = 3)$ gauge theory. Middle and Right: Pole masses m_p as a function of a bare fermion mass m_f for various IR cutoffs ($\Lambda_{\text{IR}}/\Lambda \in [10^{-5}, 3.0]$, blue-thin-solid lines) with the UV cutoff $\Lambda_{\text{UV}}/\Lambda = 10$. in the chiral broken phase: (Middle panel) $N_f = 9$, (Right) $N_f = 11$.

we derive the SD-based finite-size hyperscaling formula (FSS), which allows us to handle the IR cutoff artifacts, even in the case of $\Lambda_{\text{IR}} \gtrsim m_p, f_\pi$. These are the advantages to the previous work [3].

III. CHIRAL BROKEN AND CONFORMAL PHASES WITH IR CUTOFF

First, we investigate the chiral broken phase in the presence of IR cutoffs. The middle and right panels of Fig. 1 show the pole mass m_p against m_0 for $N_f = 9$ and 11, respectively. Each blue-thin-solid line represents a result with a different IR cutoff $\Lambda_{\text{IR}}/\Lambda$ ¹. For a small $\Lambda_{\text{IR}}/\Lambda$, the pole masses remain finite at small m_0 , indicating the spontaneous chiral symmetry breaking. We find a critical value of the IR cutoff above which the dynamical fermion mass m_D is strongly suppressed as shown in the figures:

$$\Lambda_{\text{IR}}^* = m_p(\Lambda_{\text{IR}} = 0, m_0 = 0) =: m_D. \quad (3)$$

Thus, a *Fake Conformality* results from a large Λ_{IR} . As a system becomes closer to the conformal window ($N_f = 9 \rightarrow 11$), the fake conformality appears for a smaller Λ_{IR} . This gives a caveat for lattice works studying the border region of chiral broken and conformal phases.

Next, we investigate a conformal phase with IR cutoffs by using the SD results for $N_f = 12$. The mass anomalous dimension γ_m is obtained by the scaling property of $m_p(f_\pi)$ against m_0 . The left panel of Fig. 2 shows a logarithm plot of the pole mass m_p as a function of fermion bare masses

¹ The scale Λ denotes the intrinsic scale which is renormalization invariant and associated with the onset of non-perturbative dynamics [4].

m_0 for various IR cutoffs Λ_{IR} . For the smallest IR cutoff $\Lambda_{\text{IR}} = 10^{-8}$ (red-squares), the data points align on a straight line, therefore, the hyperscaling relation $m_p = C m_0^{1/(1+\gamma_m)}$ is satisfied. For the fit range of $m_0 \in [0.001, 0.1]$ (non-shaded region in the figure), we obtain $\gamma_m = 0.56$. While, for larger IR cutoffs, the data points are on a polygonal line with a bend around $\Lambda_{\text{IR}} \simeq m_p$. As seen in the figure, the fit range $m_0 \in [0.001, 0.1]$ starts affected by the bend down with increasing Λ_{IR} . As a result, the fit with the ansatz $m_p = C m_0^{1/(1+\gamma_m)}$ results in a γ_m strongly suppressed, as shown in the middle panel of Fig. 2. When the suppression sets in, the γ_m evaluated by m_p (red-squares) and f_π (blue-circles) get impaired, and thus the universality of γ_m does not hold any more.

IV. FINITE SIZE HYPERSCALING (FSS) AND MASS ANOMALOUS DIMENSION

We shall now derive a SD-based FSS which allows us to handle the IR cutoff artifacts explained above. To this end, we further approximate the SD equation (1) with a linearized fermion propagator and $\alpha(\mu) \rightarrow \alpha_* \theta_{\Lambda-\mu}$ where α_* denotes the running coupling at IRFP, and obtain the analytic expression of the renormalization factor $Z_M^{-1} := m_0/m_p$. The Z_M is a function of dimensionless variables, $\hat{m}_p := m_p/\Lambda$, $\hat{m}_0 := m_0/\Lambda$, $\hat{\Lambda} := \Lambda_{\text{IR}}/\Lambda$, and $\gamma = 1 - \sqrt{1 - \alpha_*/\alpha_{\text{cr}}}$, and the analytic expression allows us to derive the SD-based FSS formula [7]:

$$\hat{m}_p/\hat{\Lambda}_{\text{IR}} = C_X \cdot X, \quad X \equiv \hat{m}_0^{1/(1+\gamma_m)}/\hat{\Lambda}_{\text{IR}}, \quad \Lambda_{\text{IR}} \ll m_p \ll \Lambda, \quad (4)$$

$$\hat{m}_p/\hat{\Lambda}_{\text{IR}} = C_Y \cdot Y, \quad Y \equiv \hat{m}_0/\hat{\Lambda}_{\text{IR}}^{1+\gamma_m}, \quad m_p \ll \Lambda_{\text{IR}} \ll \Lambda. \quad (5)$$

The equation (4) is same as what dictated by the renormalization group equation [5]. Remarkably, the SD predicts another FSS formula (5) which is characterized by the scaling variable Y rather than X for larger Λ_{IR} .

We apply the SD-based FSS (4) and (5) for the data obtained by solving Eq. (1) numerically. Right panel of Fig. 2 shows the results. The various symbols (colors) show data with different IR cutoffs. For the case of $\Lambda_{\text{IR}} \ll m_p \ll \Lambda$, we adopt the ansatz (4). The fit works well (blue-solid line) and we obtain $\gamma_m \simeq 0.60 =: \gamma_1$ which is fairly consistent to $\gamma_m = 0.56$ obtained in the previous section for the smallest IR cutoff $\hat{\Lambda}_{\text{IR}} = 10^{-8}$. We find the alignment of the data points, indicating the universal nature of $\gamma_m = 0.60$. For the case of $m_p \ll \Lambda_{\text{IR}} \ll \Lambda$, we adopt the ansatz (5). The fit works well (black-solid line) and gives $\gamma_m \simeq 0.56 =: \gamma_2$, which is somewhat smaller than $\gamma_1 = 0.60$ but the strong suppression has disappeared. Thus, the right FSS formula in the right place recovers the approximate universality $\gamma_1 \simeq \gamma_2$, or equivalently, the approximate data alignments in the whole mass region including both $\Lambda_{\text{IR}} \ll m_p \ll \Lambda$ and $m_p \ll \Lambda_{\text{IR}} \ll \Lambda$. Then,

two scaling variables, $X(\gamma_1)$ and $Y(\gamma_2)$, are responsible for two slopes of the alignments. In lattice studies, indeed, the formula (4) is widely used in lattice study, but the formula (5) is rarely used [6]. However, these result are of the case without paying attention to the scope of application, and opposed to each other [2].

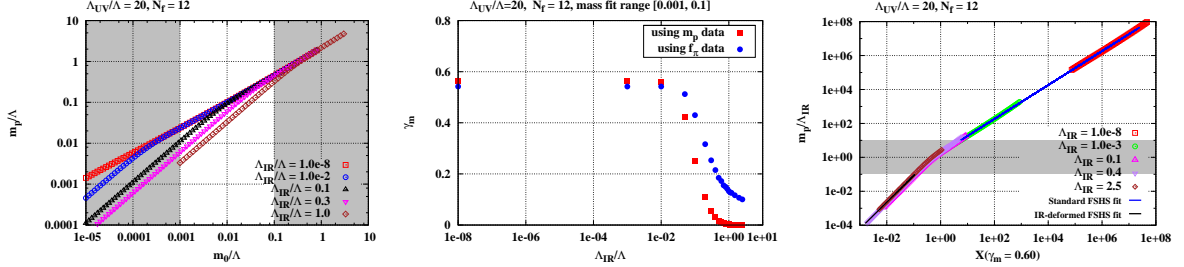


FIG. 2: Left: The pole mass $\hat{m}_p = m_p/\Lambda$ vs the fermion mass $\hat{m}_0 = m_0/\Lambda$ for various IR cutoffs $\hat{\Lambda}_{\text{IR}} = \Lambda_{\text{IR}}/\Lambda$ in $N_f = 12$. Middle: The mass anomalous dimension γ_m obtained by fitting the non-shaded region data of the left panel with the conformal ansatz. Right: The pole mass \hat{m}_p vs $X = \hat{m}_0^{1/(1+\gamma_m)}/\hat{\Lambda}_{\text{IR}}|_{\gamma_m=0.60}$ in the conformal phase $N_f = 12$. The data upper and lower than the shaded region are used for the fits, giving the blue- and black-solid lines.

Acknowledgments

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